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A New Radiation Chart

HELGI NIILISK

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TRANSLATION OF

A NEW RADIATION CHART

(Novaia radiatsionnaia nomogramma)

by

Helgi Niilisk

Akademiia Nauk Estonskoi SSR. Izvestiia, Seriia
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A NEW RADIATION CHART

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At present, graphic methods of computing the fluxes of the thermal radiation of the atmosphere with the aid of radiation charts are well known. The charts of F. Brooks [1], A. A. Dmitriev [2], R. Mügge and F. Moller [3, 4], G. Robinson [5, 6], F. N. Shekhter [7, 8], W. Elsasser [9], and G. Yamamoto [10] have been the most widely used. Unfortunately, all these charts contain certain deficiencies [11-14], viz., the use of insufficiently reliable characteristics of the absorption of long-wave radiation by atmospheric gases and the analysis of the integral function dependent only on the effective content of water vapor. The effect of carbon dioxide on absorption, moreover, is considered to be very approximate.[1] The charts mentioned do not at all consider the absorption of thermal radiation by atmospheric ozone.

It has been established [13, 14] that one of the most important factors which determines the value of radiation fluxes, calculated from any chart, is the transmission function. Therefore, to make radiation charts more accurate, it is first of all necessary to study the available quantitative characteristics of long-wave radiation in the atmosphere, and on that basis to obtain the most reliable transmission function.

Proceeding from what has been said above, the purpose of our work is to construct the integral transmission function of the atmosphere by considering the mutual effect of water vapor, carbon dioxide, and ozone[2] on the absorption of thermal radiation in the atmosphere. On the basis of this new transmission function, it is possible to solve the problem of constructing a radiation chart which is sufficiently accurate and convenient from a practical point of view. This work completes a number of studies made by the author [14-18] which were devoted to the problem of calculating the fluxes of thermal radiation in the atmosphere.

Quantitative data on the absorption of long-wave radiation, used in the present work to determine the integral transmission function, are, for the most part, given for 300°K, and also for the usual room temperatures of the order of 285-295°K. Since at surface temperatures the integral transmission function is slightly dependent on temperature [10, 13], we can consider the transmission function obtained on the basis of the mentioned data to be

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1) Consideration of the mutual influence of H_2O and CO_2 on the absorption of thermal radiation in the atmosphere is correct in principle only in G. Yamamoto's chart.

2) As is known [13], the remaining atmospheric gases play an insignificant role in the absorption of long-wave radiation by the atmosphere.

sufficiently reliable for the temperature interval 270-310°K. It should be noted that the integral transmission function was not calculated for the entire spectrum (0-∞ μ), and for the 2.27-250 μ region (in parts of the spectrum, 0-2.27 and 250-∞ μ, the amount of energy of thermal radiation is negligible and, consequently, practically has no effect on the value of the transmission function at atmospheric temperatures).

The determination of the quantitative characteristics of radiation absorption by water vapor is the subject of numerous works [1, 12, 13, 20-42], the results of which were obtained either by theoretical calculations or on the basis of an experiment (under natural or laboratory conditions). Let us note that there are important discrepancies in the data of various authors for almost all the regions of the spectrum. The results of the most recent and complete works are used when possible to determine the integral transmission function in the present work.

The values of the absorption of radiation by water vapor in the near infrared region (spectral intervals: 2.27-2.99, 2.99-3.57 and 4.88-8.7 μ) are calculated on the basis of the corresponding formulas of D. Howard, D. Burch, and D. Williams [20]. There are no experimental data for the 3.57-4.88 μ region. It is only known that in this region of the spectrum, there are no bands of absorption by water vapor and the absorption is caused by the overlapping of this interval by the wings of the absorption bands situated in the neighboring regions of the spectrum. We can assume that the logarithmic coefficient of absorption k_w here does not exceed 0.5 cm^2/g. Calculations have shown that a change in the absorption coefficient in the limits 0-0.5 cm^2/g has almost no effect on the value of the transmission function 2.27-8.7 μ, and on the value of the integral transmission function. Using this as a basis, in the present work the transmission function P for the interval 3.57-4.88 μ was calculated from the formula

$$P = e^{-k_w w^*}, \qquad (1)$$

using a certain average value $k_w = 0.2\ cm^2/g$. (here w* - the effective content of water vapor).

To determine the transmission function in the 8.7-12 μ region of the spectrum, there are no satisfactory reasons to prefer the results of one or other author (works [12, 13, 21, 22, 26, 28, 35, 37-40]). Therefore, in the present work the transmission function for the mentioned interval of the spectrum was calculated from formula (1), using certain average values for the coefficients of absorption by water vapor (Table 1).

TABLE 1

The Absorption Coefficients of Water Vapor for the 8.7-12 μ Region of the Spectrum

Spectrum Region	k_w, cm^2/g
8.7 - 9.0	0.15
9.0 - 11.5	0.10
11.5 - 12.0	0.20

Let us note that according to the most recent data [37-39], k_w, about 9.6 and 11.1 μ, is approximately 0.1.

In the 9.0-10.3 μ interval, it is necessary to consider the effect of a strong absorption band of ozone. We previously found the transmission function for the 9.0-10.3 μ region [16].

In the 12-18 μ region, the gases which absorb the thermal radiation of the atmosphere are water vapor and carbon dioxide, since a strong absorption band of carbon dioxide[1)] is found in this interval. The transmission function for 12-18 μ region obtained in works [15, 18] on the basis of data [19] and [22] is used in the present work.

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1) According to [13] we can disregard the effect of the remaining bands of absorption by carbon dioxide on the transfer of long-wave radiation in the atmosphere.

Data are given in works [13, 21, 22, 27-30, 36, 41, 42] on the absorption of long-wave radiation by water vapor in the far infrared region of the spectrum. A comparison of the results of these works shows that the results of the theoretical calculations of G. Yamamoto [22] coincide most closely with the most recent data of the laboratory measurements of C. Palmer [36]. Proceeding from this, the transmission function for the 18-250 μ interval in the present work is determined on the basis of the usage of generalized coefficients of absorption by water vapor according to the data of work [22].

Let us note that for all the examined intervals of the spectrum, the transmission function for diffuse radiation was determined from the following known relationship [13]:

$$P_F(w^*) = 2\int_0^{\pi/2} P(w^* \sec\theta)\sin\theta\cos\theta\, d\theta, \qquad (2)$$

where P_F is the transmission function for diffuse radiation, P is the transmission function for the directed radiation, θ is the zenith angle.

By using the results of the determination of the transmission functions for various parts of the spectrum, the integral transmission function for diffuse radiation was calculated from the following formula:

$$P_F = \frac{1}{f}\sum_{\Delta} f_\Delta P_{F\Delta}. \qquad (3)$$

Here the index Δ designates the examined intervals of the spectrum, f is a fraction of the integral radiation of an absolutely black body arriving on the 2.27-250 μ part of the spectrum, f_Δ is a fraction of the radiation of an absolutely black body arriving at the interval of the spectrum Δ.

Tables 2 and 3 give the calculation results. The tables also give the values of the functions $\Delta P_1 (w^*, u^*)$ and $\Delta P_2 (w^*, m^*)$ with which the sought integral transmission function can be calculated from the relationship:

$$P_F(w^*, u^*, m^*) = 0{,}001\,(\Delta P_1 + \Delta P_2). \qquad (4)$$

where u* is the effective content of carbon dioxide, m* is the effective content of ozone.[1]

Since during the compilation of Tables 2 and 3 approximate methods were also used (numerical integration, interpolation), we can consider the transmission function obtained reliable to an accuracy of 1/100.

Having the data of an aerological sounding, it is easy to determine the values of the fluxes of thermal radiation in the atmosphere by using Tables 2 and 3 and the usual graphic method based on the use of relationships (see [13]):

$$G = \int P_F \, dB, \tag{5}$$

where G is the flux of thermal radiation of the atmosphere, $B = \sigma T^4$ is the integral flux of radiation of an absolutely black body.

In other words, G is numerically equal to the area in the coordinate system (P_F, B).

Figure 1 gives the chart for the determination of G.

It should be not ed that formula (5) was derived on the assumption of the independence of the integral transmission function on temperature [13]. But, as is known [2, 4, 9, 10, 13], the transmission function actually depends not only on the content of absorbing matter in the atmosphere, but also on pressure and temperature. The dependence P_F on pressure is usually calculated with the aid of an effective absorbing mass, (see [1-10, 13, 14]). However, the problem of calculating the temperature dependence has not yet been finally solved, and, to a considerable degree, this problem may be considered debatable [13].

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1. The values w*, u*, and m* are expressed in "cm" (the thickness of the layer of the precipitated matter in centimeters under normal pressure and temperature).

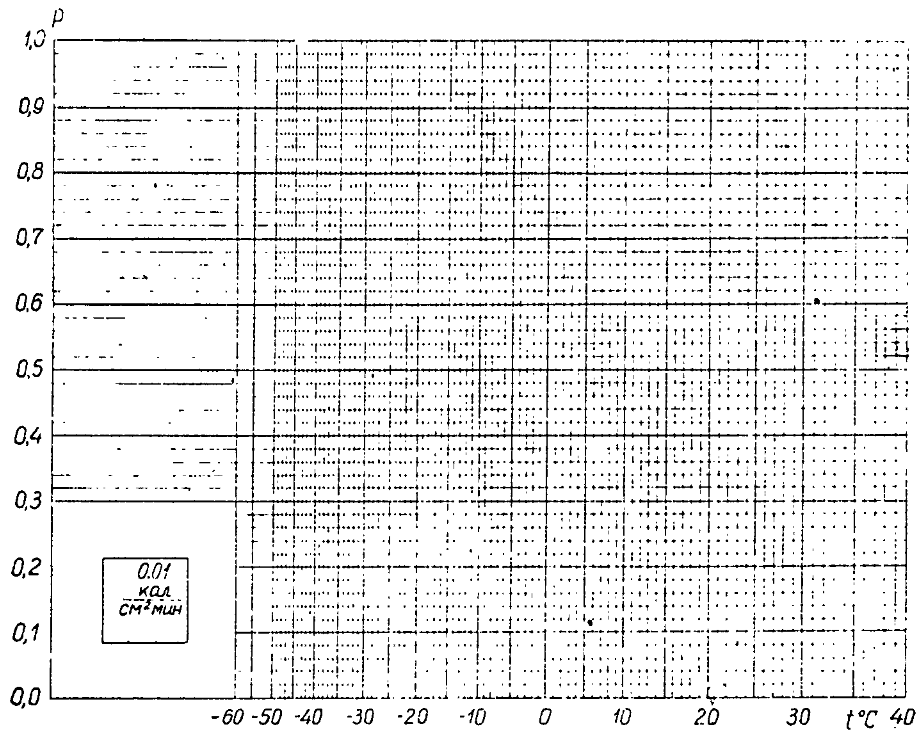

Fig. 1. Chart for the determination of fluxes of long-wave radiation in the atmosphere.

With a radiation chart, the present work attempts to evaluate the effect of the dependence $P_F = P_F(T)$ on the determination of the fluxes of the thermal radiation of the atmosphere.

By solving the general equations for the transfer of long-wave radiation in the atmosphere, we have the following expression for the intensity of the descending monochromatic radiation which propagates in the direction θ [13]:

$$I_\lambda \downarrow (z, \theta) = \int_z^\infty \frac{k_\lambda(T,p)}{\cos\theta}\, \rho(\eta) E_\lambda(T)\, e^{-\frac{1}{\cos\theta}\int_z^\eta k_\lambda(T,p)\rho(\xi)\, d\xi}\, d\eta. \qquad (6)$$

Here $I_\lambda \downarrow (z, \theta)$ - is the intensity of the descending long-wave monochromatic radiation at the level z, k_λ (T, p) is the absorption coefficient for the wave length λ, T = T (η) is the temperature of the absorbing (radiating)

medium, $p = p(\eta)$ is the total pressure in the atmosphere, ρ is the density of the matter absorbing the radiation, $E_\lambda(T)$ is the intensity of the radiation of an absolutely black body for wave-length λ, θ is the zenith angle.

Let us express the absorption coefficient in the following form:

$$k_\lambda(T, p) = k_{o\lambda} F(T, p), \tag{7}$$

where $F(T, p)$ is a certain function of temperature and pressure ($k_{o\lambda}$ is considered independent of T and p).

With the calculation of (7), expression (6) will have the following form:

$$I_\lambda\downarrow = \int_z^\infty \frac{k_{o\lambda}}{\cos\theta} F(T,p)\,\varrho(\eta)\,E_\lambda(T)\,e^{-\frac{k_{o\lambda}}{\cos\theta}\int_z^\eta F(T,p)\,\varrho(\xi)\,d\xi}\,d\eta. \tag{8}$$

Let us introduce the effective absorbing mass w* which is determined in the following fashion:

$$w^* = \int_z^\eta F(T,p)\,\varrho(\xi)\,d\xi. \tag{9}$$

In such a case

$$I_\lambda\downarrow = \int_0^{w^*_\infty} \frac{k_{o\lambda}}{\cos\theta} E_\lambda(T)\,e^{-\frac{k_{o\lambda}}{\cos\theta}w^*}\,dw^*. \tag{10}$$

Integrating (10) according to λ with the aid of the Ambartsumian-Lebedinskii method [13], we have

$$I\downarrow = \int_0^{w^*_\infty} E(T)\,dw^* \int_0^\infty \frac{k_0}{\cos\theta} f(k_0, T)\,e^{-\frac{k_0}{\cos\theta}w^*}\,dk_0. \tag{11}$$

It was assumed here that in regions of the spectrum, for which the inequality $k_o < k_{o\lambda} < k_o + dk_o$, the value of the absorption coefficient may in practice be considered as constant (k_o). The function $f(k_o, T)$ determines a

TABLE 2

Values of the function $\Delta P_1 (w^*, u^*)$

lg u^* / lg w^*	$\bar{2}$,00	$\bar{1}$,00	$\bar{1}$,20	$\bar{1}$,40	$\bar{1}$,60	$\bar{1}$,80	0,00	0,10	0,20	0,30	0,40	0,50	0,60	0,70	0,80	0,90	1,00	1,10	1,20	1,30	1,40	1,50	1,60	1,70	1,80	1,90	2,00	2,10	2,20	2,30	2,40	2,50
$\bar{4}$,00	868	851	848	841	832	823	812	806	800	793	787	781	774	768	761	755	749	744	739	733	727	722	717	711	706	701	696	691	686	682	677	673
$\bar{4}$,10	861	850	844	837	828	819	808	802	796	789	783	777	770	764	757	751	745	740	735	729	723	718	713	707	702	697	692	687	682	678	673	669
$\bar{4}$,20	858	844	838	831	822	813	802	796	790	783	777	771	764	758	751	745	739	734	729	723	717	712	707	701	696	691	686	681	676	672	667	663
$\bar{4}$,30	853	838	833	825	816	807	796	790	784	777	771	765	758	752	745	739	733	728	723	717	711	706	701	695	690	685	680	675	670	666	661	657
$\bar{4}$,40	845	831	825	818	809	800	789	783	777	770	764	758	751	745	738	732	726	721	716	710	704	699	694	688	683	678	673	668	663	659	654	650
$\bar{4}$,50	838	823	817	810	801	792	781	775	769	762	756	750	743	737	730	724	718	713	708	702	696	691	686	680	675	670	665	660	655	651	646	642
$\bar{4}$,60	829	815	809	802	793	784	773	767	761	754	748	742	735	729	722	716	710	705	700	694	688	683	678	672	667	662	657	652	647	643	638	634
$\bar{4}$,70	820	806	800	794	784	775	764	758	752	745	739	733	726	720	713	707	701	696	691	685	679	674	669	663	658	653	648	643	638	634	629	625
$\bar{4}$,80	811	797	791	784	775	766	755	749	743	736	730	724	717	711	704	698	692	687	682	676	670	665	660	654	649	644	639	634	629	625	620	616
$\bar{4}$,90	800	786	780	773	764	755	744	738	732	725	719	713	706	700	693	687	681	676	671	665	659	654	649	643	638	633	628	623	618	614	609	605
$\bar{3}$,00	786	773	767	760	751	742	731	725	719	712	705	700	693	687	680	674	668	663	658	652	646	641	636	630	625	620	615	610	605	601	596	592
$\bar{3}$,10	774	760	754	747	738	729	718	712	706	699	693	687	680	674	667	661	655	650	645	639	633	628	623	617	612	607	602	597	592	588	583	579
$\bar{3}$,20	759	746	740	733	721	715	704	698	692	685	679	673	666	660	653	647	641	636	631	625	619	614	609	603	598	593	588	583	578	574	569	565
$\bar{3}$,30	743	730	724	716	707	698	687	681	675	668	662	656	649	643	636	630	624	619	614	608	602	597	592	586	581	577	572	567	562	558	553	549
$\bar{3}$,40	728	714	708	701	692	683	672	666	660	653	647	641	634	628	621	615	609	604	599	593	587	582	577	571	566	562	557	552	547	543	538	534
$\bar{3}$,50	714	700	694	687	678	669	658	652	646	639	633	627	620	614	607	601	595	590	585	579	573	568	563	557	552	548	543	538	533	529	524	520
$\bar{3}$,60	700	686	680	673	664	655	644	639	633	626	619	613	606	600	593	588	582	577	572	566	560	555	550	544	539	535	530	525	520	516	511	507
$\bar{3}$,70	687	673	667	660	651	642	631	626	620	613	606	600	593	587	580	575	569	564	559	553	547	542	537	531	526	522	517	512	507	503	498	494
$\bar{3}$,80	674	660	654	647	638	629	619	613	607	600	594	587	580	574	568	562	556	551	546	540	534	529	524	519	514	509	504	499	494	490	486	482
$\bar{3}$,90	661	647	641	634	625	615	606	600	594	587	581	574	567	561	555	549	543	538	533	527	521	516	511	506	501	496	491	486	481	477	473	469
$\bar{2}$,00	648	631	628	621	612	603	593	587	581	574	568	561	555	548	542	537	531	526	521	515	509	504	499	494	489	484	479	474	469	465	461	457
$\bar{2}$,10	634	620	614	607	599	591	580	574	568	561	555	548	542	535	529	523	517	512	507	501	495	491	486	481	476	471	466	461	456	452	447	443
$\bar{2}$,20	621	607	601	594	585	577	567	561	555	548	541	534	528	522	516	510	504	499	494	488	483	478	473	468	463	458	453	448	443	439	434	430
$\bar{2}$,30	607	593	587	580	572	563	553	547	541	535	528	521	515	508	502	497	491	485	481	475	469	464	459	455	449	444	439	434	430	426	421	417
$\bar{2}$,40	592	578	572	566	558	548	538	533	527	520	514	507	500	494	488	483	477	472	467	461	456	451	446	442	437	431	426	422	416	412	408	404
$\bar{2}$,50	578	564	558	551	544	535	525	520	514	507	500	493	487	481	475	470	464	459	454	448	443	438	433	428	423	418	413	408	403	399	394	390
$\bar{2}$,60	562	549	543	537	529	520	510	505	498	492	485	479	473	467	461	456	450	445	440	434	429	424	419	414	409	404	399	395	391	387	382	378
$\bar{2}$,70	545	532	526	520	512	503	494	489	483	476	469	463	457	451	445	440	435	430	425	419	414	409	404	399	394	390	385	380	376	372	367	363
$\bar{2}$,80	529	516	510	504	496	487	478	473	467	461	454	448	442	436	430	425	420	415	410	404	399	395	390	386	381	376	371	367	362	358	354	350
$\bar{2}$,90	511	498	493	487	479	471	461	456	450	444	438	432	426	421	415	410	405	400	395	389	384	380	375	371	366	361	357	353	348	344	340	336
$\bar{1}$,00	492	490	475	469	461	453	444	438	433	427	421	415	409	404	398	393	388	383	378	373	368	364	359	355	350	346	342	338	333	329	326	322
$\bar{1}$,10	472	460	451	448	441	433	425	419	415	409	403	397	392	387	382	377	372	367	362	357	352	348	343	339	335	331	327	323	318	314	311	307
$\bar{1}$,20	452	440	435	429	422	414	405	401	396	390	385	380	371	369	365	360	355	350	345	341	336	332	327	323	320	316	312	308	304	300	297	294
$\bar{1}$,30	428	417	410	407	401	392	384	379	375	370	365	359	351	349	345	340	335	330	326	322	318	315	310	305	303	299	296	293	289	285	282	279
$\bar{1}$,40	405	394	390	384	378	371	364	359	354	349	345	340	335	330	326	321	316	311	307	304	301	297	293	290	287	283	280	277	273	270	267	264
$\bar{1}$,50	380	369	365	360	354	347	340	333	332	328	323	319	315	310	306	302	297	293	289	285	282	279	276	272	270	265	263	260	255	253	250	246
$\bar{1}$,60	352	344	338	332	327	320	314	311	307	305	301	297	294	289	285	281	278	274	270	267	264	261	258	255	253	250	247	244	240	237	234	230
$\bar{1}$,70	322	312	308	304	299	293	288	285	282	280	277	271	270	267	264	260	256	253	250	247	244	242	240	238	235	233	230	227	224	221	219	217
$\bar{1}$,80	292	283	280	277	271	264	261	261	258	255	253	250	247	245	242	239	235	233	230	227	225	221	222	220	218	215	212	209	207	205	203	201
$\bar{1}$,90	262	255	253	249	246	242	238	236	233	230	228	226	221	222	218	215	213	211	209	207	205	203	202	201	197	[illegible]	196	191	189	187	185	183
0,00	231	226	225	221	219	216	212	210	209	206	203	202	200	198	196	194	192	190	188	185	184	182	180	179	177	176	174	173	171	169	168	167
0,10	201	197	195	193	190	187	185	184	182	180	178	176	174	173	172	170	168	167	165	163	161	160	159	158	157	156	155	154	152	151	150	148
0,20	170	167	166	164	162	160	158	157	155	154	153	151	150	149	148	147	146	145	144	143	142	141	140	139	138	137	136	135	134	133	132	131
0,30	141	139	138	137	135	134	133	132	131	130	129	128	127	127	126	126	125	125	124	123	122	121	120	119	118	118	117	117	116	115	114	113
0,40	115	115	114	113	112	111	110	109	108	108	107	107	106	105	105	105	104	104	104	103	103	103	102	102	101	101	100	100	99	98	97	97
0,50	91	91	90	90	90	89	89	88	88	88	87	87	87	86	86	86	85	85	84	84	84	84	83	83	83	83	82	82	82	81	81	81
0,60	72	72	72	72	71	71	71	70	70	70	70	70	70	69	69	69	69	69	69	69	69	68	68	68	68	68	67	67	67	67	67	66
0,70	56	56	56	56	56	56	56	55	55	55	55	55	55	55	55	55	55	55	55	55	54	54	54	54	54	54	53	53	53	53	53	53
0,80	43	43	43	43	43	43	43	43	43	43	43	43	43	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42
0,90	32	32	32	32	32	32	32	32	32	32	32	32	32	32	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31
1,00	23	23	23	23	23	23	23	23	23	23	23	23	23	23	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22

TABLE 3

Values of the function ΔP_1 (w*, m*)

lg m / lg w*	$\bar{4},00$	$\bar{3},00$	$\bar{3},50$	$\bar{2},00$	$\bar{2},10$	$\bar{2},20$	$\bar{2},30$	$\bar{2},40$	$\bar{2},50$	$\bar{2},60$	$\bar{2},70$	$\bar{2},80$	$\bar{2},90$	$\bar{1},00$	$\bar{1},10$	$\bar{1},20$	$\bar{1},30$	$\bar{1},40$	$\bar{1},50$	$\bar{1},60$	$\bar{1},70$	$\bar{1},80$	$\bar{1},90$	0,00
$\bar{4},00$	87	87	86	83	82	81	80	78	76	73	71	68	65	62	58	55	51	48	45	42	40	38	36	34
$\bar{2},00$	87	87	86	83	82	81	80	78	76	73	71	68	65	62	58	55	51	48	45	42	40	38	36	34
$\bar{1},00$	86	85	84	81	80	79	78	76	74	72	69	67	64	61	57	53	50	47	44	41	39	37	35	32
$\bar{1},20$	84	84	83	80	79	78	77	76	74	72	69	67	64	62	57	53	50	47	44	41	39	37	35	32
$\bar{1},40$	82	82	82	79	78	77	76	75	73	71	68	66	63	59	56	52	49	46	43	40	38	36	34	32
$\bar{1},60$	80	80	80	77	76	75	74	73	71	69	66	64	61	57	54	51	48	45	42	39	37	35	33	31
$\bar{1},80$	77	77	77	75	74	73	71	69	67	65	63	62	60	55	52	49	46	44	41	38	36	34	33	31
84	76	76	75	74	74	73	71	69	67	65	63	62	58	54	51	48	45	44	41	38	36	34	32	30
88	76	75	75	72	73	72	70	68	66	64	62	61	58	54	51	48	45	43	40	37	35	33	32	30
92	75	74	74	71	71	70	69	67	65	63	61	60	57	54	50	47	44	42	39	37	35	34	31	29
$\bar{1},96$	74	73	73	70	70	69	68	66	65	63	61	59	56	52	49	46	43	41	38	36	34	32	31	29
0,00	73	72	72	69	68	68	67	65	64	62	60	58	55	51	48	45	43	40	38	35	34	32	30	28
0,04	72	71	70	68	68	67	66	64	63	61	59	57	54	50	47	44	42	39	37	35	33	31	30	28
0,08	70	70	69	67	67	66	65	63	62	60	58	56	53	49	46	43	41	38	36	34	33	31	29	27
0,12	69	69	68	66	66	65	64	62	61	59	57	55	52	48	45	42	40	37	35	34	32	30	29	27
0,16	67	67	66	64	64	63	62	61	60	58	56	54	51	47	44	41	39	36	34	33	32	30	28	26
0,20	66	66	65	63	63	62	61	60	59	57	55	53	50	46	43	40	38	36	34	32	31	29	28	26
0,24	64	64	63	61	61	60	59	58	57	55	54	52	49	45	42	39	37	35	33	31	30	29	27	25
0,28	62	62	61	59	59	58	57	56	55	54	52	50	47	44	41	38	36	34	32	30	29	28	26	24
0,32	60	60	59	57	57	56	55	54	53	52	51	49	46	42	40	37	35	33	31	29	28	27	25	23
0,36	58	58	57	56	56	55	54	53	52	51	50	48	45	41	39	36	34	32	30	28	27	26	25	23
0,40	56	56	55	54	53	53	52	50	49	48	47	45	43	40	38	35	33	31	29	27	26	25	24	22
0,44	54	54	53	52	52	52	50	49	48	47	46	44	41	38	36	34	32	30	28	26	25	24	24	21
0,48	52	52	51	50	50	49	48	47	46	44	43	42	39	37	35	33	31	29	27	25	24	24	22	20
0,52	50	50	49	47	47	46	45	44	43	42	41	40	37	35	33	31	29	28	26	24	23	22	21	19
0,56	48	48	47	45	45	44	43	42	41	40	39	38	36	34	32	30	28	27	25	23	22	21	20	18
0,60	45	45	44	43	43	42	41	40	39	38	37	36	34	32	30	28	26	25	24	22	21	20	19	17
0,64	43	43	42	41	41	40	39	38	38	37	36	35	33	31	29	27	25	24	22	21	20	19	18	16
0,68	40	40	39	38	38	37	37	36	35	35	34	33	31	29	28	26	24	23	21	20	19	18	17	15
0,72	37	37	37	36	36	35	34	34	33	32	31	30	29	27	26	24	22	21	20	19	18	17	16	14
0,76	34	34	34	33	33	32	32	31	31	30	29	28	27	25	24	23	21	20	18	17	17	16	15	13
0,80	31	31	31	30	30	29	29	28	28	27	27	26	25	23	22	21	19	18	16	15	15	14	13	12
0,84	28	28	28	27	27	27	26	26	26	25	25	24	23	22	20	19	17	16	15	14	13	13	12	11
0,88	25	25	25	25	25	25	24	24	23	23	22	21	21	20	19	18	16	15	14	13	12	12	11	10
0,92	23	23	23	23	23	23	22	22	21	21	20	19	19	18	17	16	15	14	13	12	11	11	10	9
0,96	21	21	21	20	20	20	19	19	19	18	18	17	17	16	15	14	13	12	11	11	10	10	9	8
1,00	19	19	19	18	18	18	17	17	17	16	16	15	15	14	13	12	11	10	10	9	9	8	8	7

fraction of the intensity of an absolutely black body arriving on the total number of regions of the spectrum to which the absorption coefficient k_o corresponds. E(T) in expression (11) designates the integral intensity of the radiation of an absolutely black body.

On the other hand, it is easy to show that by using an effective absorbing mass which is determined from formula (9), the integral transmission function of the atmosphere is expressed as follows:

$$P(w^*, T_1) = \int_0^\infty f(k_0, T_1) e^{-\frac{k_0}{\cos\vartheta} w^*} dk_0. \tag{12}$$

Here T_1 designates the temperature of the source of the radiation; and it is assumed that this source is an absolutely black body.

In addition, let us introduce a new function P* which is determined by the relationship

$$P^*[w^*, T(w^*)] = P[w^*, T_1 = T(w^*)]. \tag{13}$$

In other words,

$$P^*[w^*, T(w^*)] = \int_0^\infty f[k_0, T(w^*)] e^{-\frac{k_0}{\cos\vartheta} w^*} dk_0. \tag{14}$$

Now let us find the derivatives P and P* with respect to w*:

$$\frac{dP}{dw^*} = -\int_0^\infty \frac{k_0}{\cos\vartheta} f(k_0, T_1) e^{-\frac{k_0}{\cos\vartheta} w^*} dk_0. \tag{15}$$

Here it is assumed that temperature T_1 does not change.

$$\frac{dP^*}{dw^*} = -\int_0^\infty \frac{k_0}{\cos\vartheta} f(k_0, T) e^{-\frac{k_0}{\cos\vartheta} w^*} dk_0 + \frac{dT}{dw^*} \int_0^\infty \frac{\partial f(k_0, T)}{\partial T} e^{-\frac{k_0}{\cos\vartheta} w^*} dk_0. \tag{16}$$

We see that the integral $S_1 = \int_0^\infty \frac{k_0}{\cos\vartheta} f(k_0, T) e^{-\frac{k_0}{\cos\vartheta} w^*} dk_0$ in expression (11) is equal to the first term of expression (16). Therefore, using dP*/dw* and dP/dw* instead of S_1, creates errors. In the first case we do not consider the second term of expression (16), and in the second case the "displacement effect" is not considered.

Let us try to make an approximate evaluation of the magnitude of these errors. Under average conditions in the troposphere[1], the relationship between T and w* is well described by the formula

$$T = -8(w^*)^8 - 10w^* + 20, \tag{17}$$

where w* is expressed in "cm" and T in C^o.

To simplify the calculations, let us use instead of expressions (12) and (14) the approximate formulas

$$P(w^*, T_1) = \sum_{j=1}^{6} f_j(T_1) e^{-k_j w^*}, \tag{18}$$

where T_1 is considered constant, and

$$P^*(w^*, T) = \sum_{j=1}^{6} f_j(T) e^{-k_j w^*}, \tag{19}$$

where T = T(w*).

Formulas (18) and (19) are taken from the monograph of K. Ia. Kondrat'ev [13].

This work also gives the absorption coefficients k_j and the connection between f_j and the temperature:

$$f_j = a_j + b_j T. \tag{20}$$

Here a_j and b_j are certain constants for each region j.

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1. It is assumed that the vertical temperature gradient is equal to 6 deg/km, and the vapor density ρ_w decreases with height according to exponential law. At the earth's surface T = 20° C and ρ_w = 7 x 10^{-6} g/cm^3.

On the basis of the data given above, the following magnitudes were calculated in the present work: S_1, S_2 = dP*/dw*, S_3 = dP(T_1 = 260°K)/dw*, S_4 = dP(T_1 = 300°K)/dw*. Figure 2 shows the results of these calculations. As this figure shows, the use of dP*/dw* instead of S_1 causes many great errors, especially in the upper layers of the troposphere. The differences between S_1 and S_3, and also between S_1 and S_4 are rather small. Therefore, it is necessary to consider as completely substantiated the use of dP(w*, T_1)/dw* instead of S_1 in formula (11) (assuming that T_1 is a certain average temperature in the atmosphere). In other words, we can disregard the influence of the displacement effect in the given case.

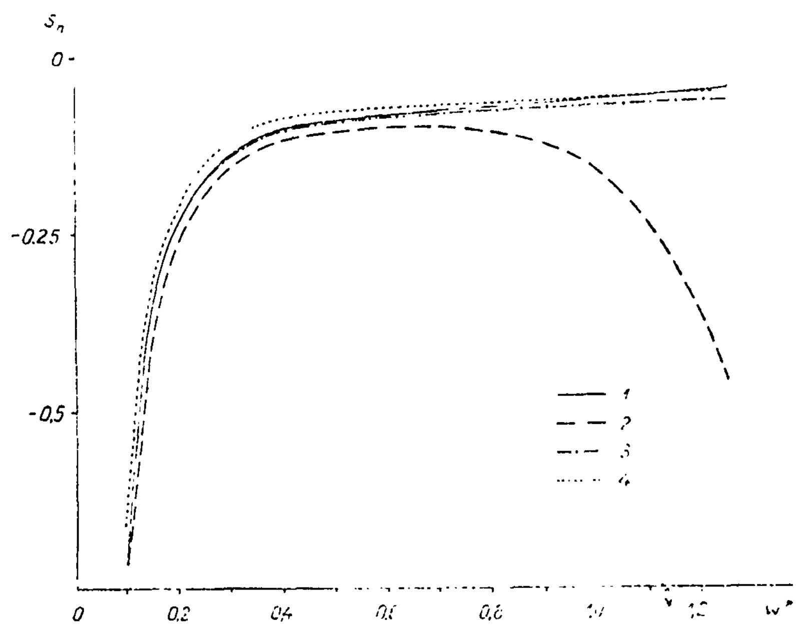

Fig. 2. A comparison of the functions S_n: 1-S_1, 2-S_2, 3-S_3, 4-S_4.

As is known [13], the dependence of the absorption coefficient on temperature and the "displacement effect" act in opposite directions. Therefore, it should be considered as proven that the relationship $k_\lambda = k_\lambda(T)$ is not considered. In such a case formula (9) has the following form:

$$w^* = \int F(p)\varrho(z)dz, \tag{21}$$

where F(p) is a certain function of pressure.

On the basis of what has been said above, instead of (11) we will get

$$I\downarrow = \int_{w^*_\infty}^{0} E(T)dw^* \frac{dP}{dw^*} = \int_{P(w^*_\infty)}^{1} E(T)dP. \tag{22}$$

Partially integrating (22), we get an expression analogous to formula (5):

$$I\leftarrow = \oint PdE. \tag{23}$$

We can obtain an expression even for I ↑ in this way.

We should note that the results given above are justified even for fluxes of long-wave radiation in the atmosphere. As is known [13], the radiation fluxes are calculated by integrating the corresponding intensities according to the solid angles corresponding to a semisphere. In such a case, we will have formula (5) instead of formula (23).

Summarizing the results given above, we can say that on the basis of the approximate formula G = $\int P_F(w^*)dB$, we can determine with sufficient accuracy the long-wave fluxes of the radiation of the atmosphere, but the use of the formula G = $\int P_F[w^*, T(w^*)]dB$ cannot be justified.

As was shown in works [16] and [43], atmospheric ozone plays only an insignificant role in the determination of the values of the integral fluxes of thermal radiation in the lower troposphere. Therefore, the effect of ozone on the absorption of long-wave radiation in the atmosphere can be disregarded when calculating descending fluxes (G ↓) in the lower troposphere and ascending fluxes (G ↑) to 15-20 km. In such a case, we can use only one table instead of Tables 2 and 3, in which the values of the integral transmission function P_F(w*, u* are given.

As an example, we calculated the descending flux, the ascending flux, and the effective radiation ($F = G\uparrow - G\downarrow$) at the levels 0. 3 and 8 km for certain latitudinal zones of the earth. In addition, stratifications of temperature, humidity, and pressure, taken from works [10, 44, 45] were used. Data on the vertical distribution of ozone were taken from work [46]. The concentration of carbon dioxide in the atmosphere was taken equal to 0. 03% (by volume) for all the zones, since there were no data on the latitudinal variation of CO_2. When calculating the effective absorbing masses of water vapor, we applied the correction which is usually used $(p/p_o)^{0.5}$, and for carbon dioxide and ozone this correction was $(p/p_o)^{0.8}$ and $(p/p_o)^{0.2}$ respectively (see [15, 16, 18]). Table 4 gives the results of these calculations.

TABLE 4

Values of the Fluxes of Thermal Radiation in the Atmosphere (cal/cm^2 min)

latitudinal zone (°N)	$z = 0$		$z = 3$		$z = 8$	
	$G\downarrow$	F	$G\downarrow$	F	$G\downarrow$	F
0—10	0,574	0,093	0,392	0,194	0,164	0,296
10—20	0,545	0,101	0,348	0,209	0,132	0,317
20—30	0,507	0,114	0,312	0,220	0,117	0,313
30—40	0,429	0,117	0,275	0,208	0,097	0,293
40—50	0,359	0,122	0,240	0,198	0,082	0,270
50—60	0,296	0,125	0,201	0,187	0,076	0,246
60—70	0,236	0,129	0,169	0,172	0,077	0,218

Analogous calculations are carried out in works [14, 17] with the aid of the charts of F. Brooks, A. A. Dmitriev, R. Mügge and F. Möller, G. Robinson, F. N. Shekhter, V. Elsasser and G. Yamamoto. Comparing the results of the determination of $G\downarrow$, $G\uparrow$ and F which were obtained in the present work and in works [14, 17], we see that near the earth's surface and in the lower part of the troposphere, our results agree rather closely with the data obtained from the charts of F. Shekhter and F. Brooks. However,

when $z \approx 8$ km, there is a closer agreement with the results of the calculations from the charts of F. Möller and G. Robinson.

Unfortunately, at present there is no satisfactorily complete complex of experimental data, which makes it impossible to make a direct comparison of the results with the experiment.

Since the radiation nomogram chart in this work was constructed on the basis of a most careful analysis of contemporary data on the absorption of infrared radiation in the atmosphere, it may be considered most reliable.

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